Breaking the Curve

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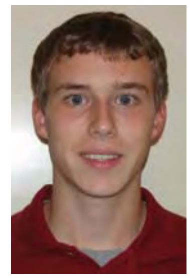


All four authors are graduates of Fairmont High School and each participated in a Dual-Enrollment Program taking Multi-variable calculus through the University of Dayton. All four authors would like to thank not only their devoted teacher, Scott Mitter but Kettering City Schools for allowing such a great program to exist. ***Amanda Dahlman*** *will be attending Wright State University majoring in math and engineering.* ***Jesse DePinto*** *will be attending Marquette University majoring in physics.* ***Kyle Kremer*** *will be attending Northwestern University majoring in trumpet performance and physics.* ***Joe Plattenburg*** *will be attending The Ohio State University majoring in mathematics and engineering.*

In this paper we will derive a model for the path of a baseball pitch based on actual data from MLB.com's *GameDay*™ feature. Then we will define the concepts of break and curvature with respect to the trajectory of a pitch. Employing the model we developed for the position of the pitch, we can calculate the curvature and break at any time during the ball's flight by deriving equations for these values. Finally, we shall analyze the extrema of the curvature and break of the pitch, coming to a conclusion on the relationship between the two.

Applying the Data

On Major League Baseball's website, there is a feature called *Gameday* which uses high speed cameras to collect data on every pitch thrown in a given game. We will use the data from this website to create our model and analyze certain pitches. All this information is readily available on MLB.com and can be viewed in Excel format. The website's data include many aspects of the pitches, but we are most concerned with the initial position, initial velocity, and acceleration of the pitches. For the purposes of our project, we assume that once the ball has left the pitcher's hand, acceleration is only due to gravity, and therefore is constant. This is a reasonable assumption because the time from which the ball leaves the pitcher's hand until the ball is caught or hit is around 0.5 second.

Before applying data, we first set up the symbolic equation for the position of any pitch. The coordinate system we will use has the back of home plate at the origin. The positive *y*-axis extends from home plate toward the pitcher's mound, the positive *x*-axis extends toward the "outside" part of the plate for a right handed batter, or in the general direction of the "first-base side" of the field, and the positive *z*-axis points straight up. We use feet and inches for our measurements of distance, and seconds for our measurements of time.

To find the symbolic equation for the position of the pitch, we need to find the velocity vector, so we integrate each component of the initial acceleration using the initial velocities as integration constants:

$$\int \mathbf{a}(t)dt = \mathbf{v}(t) = \left\langle a_x t + v_{0x},\ a_y t + v_{0y},\ a_z t + v_{0z} \right\rangle \qquad (1)$$

Then, to find the position vector, we integrate the velocity using the initial positions:

$$\int \mathbf{v}(t)dt = \mathbf{r}(t) = \left\langle \frac{1}{2}a_x t^2 + v_{0x}t + r_{0x}, \frac{1}{2}a_y t^2 + v_{0y}t + r_{0y}, \frac{1}{2}a_z t^2 + v_{0z}t + r_{0z} \right\rangle \qquad (2)$$

Now, with the general equation for the position of the ball, we use MLB.com's *Gameday* feature to find data for one of the pitches thrown by the Boston Red Sox's Daisuke Matsuzaka, namely its initial position (in feet) (–2.338, 50, 5.526); initial velocity (in feet per second) $\langle 5.383, -116.917, -0.401 \rangle$; and acceleration $\langle 7.497, 28.723, -36.663 \rangle$ (in feet per second squared). Below is an example of the excel format in which some data for some other pitches by Matsuzaka are listed. Although the actual distance from the pitcher's mound to home plate is 60.5 feet, the initial *y*-position we use is 50 ft because the data is not recorded until the ball is several feet in front of the pitcher's mound.

ax	ay	az	break_angle	break_length	break_y
-13.239	34.356	-12.745	37.6	4.1	23.7
-11.472	39.418	-14.092	32.9	4.1	23.6
-11.434	36.507	-15.881	28.9	4.4	23.6
-10.14	40.523	-13.843	27	3.7	23.6
-11.211	38.337	-16.167	27	4.4	23.6
7.633	29.615	-32.686	-14.1	9.6	23.7

Figure 1

We used a tutorial by Alan Nathan to interpret the data. By applying these data to the Equation 2, we can derive a specific position vector function for our particular pitch:

$$\boldsymbol{r}(t) = \langle 3.749t^2 + 5.383t - 2.338, 14.137t^2 - 116.917t + 50, -18.332t^2 - 0.401t + 5.526 \rangle. \qquad (3)$$

With this function we can analyze both the break and the curvature of the pitch.

Break

The first aspect of the pitch we will analyze is the *break*, which is defined as the deviation of the pitch (the actual path of the pitch, which is the upper, curved line in the diagram below) from a straight-line trajectory (the lower, straight line in the diagram) connecting the initial position to the final position. As can be seen in Figure 2, the pitch starts out at y = +50 (at r_0) and its break increases until it reaches a maximum about 2/3 of the way, then decreases until it reaches the plate (at r_f).

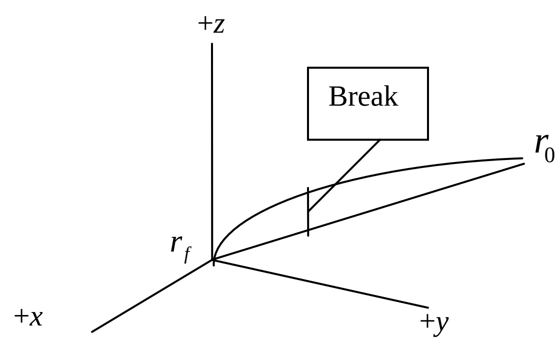
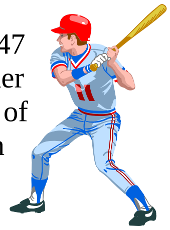


Figure 2

To analyze the pitch's break, we will need to find a function for the pitch's break at any time. To find a break function, we need a function for the position of the pitch at any time (which we already have defined) and a function for the straight-line trajectory. The straight-line trajectory vector is defined as the vector whose endpoints are at the final and initial positions of the pitch:

$$\mathbf{r_0 r_f} = r_f - r_0$$



We can find the final position by setting the y-component of the position vector equal to 1.147 feet, and then solving for time. We use 1.147 feet (this will be converted to inches later) rather than 0 because MLB.com's *GameDay* records the final speed at y = 1.147 feet (at the front of home plate). This time is 0.438 seconds, so we substitute 0.438 seconds into the position equation to find the final position of the pitch (0.739, 1.147, 1.834).

Now that we have the final and initial positions, we can find the straight-line trajectory function using the standard point-slope format. We chose the coefficients in Equation 4 so that

$\mathbf{r_0 r_f}(0.438) = \mathbf{r_f}$ and the constants so that $\mathbf{r_0 r_f}(0) = \mathbf{r_0}$.

$$\mathbf{r_0 r_f}(t) = \langle -2.338 + 7.025t, 50 - 111.537t, 5.526 - 8.432t \rangle \qquad (4)$$

With the equation for the straight-line trajectory and the equation for the position of the pitch, we can find the distance between the two functions (this is the break) at any given time.

The break function for our specific pitch is:

$$b(t) = 12\sqrt{(3.749t^2 - 1.642t)^2 + (14.137t^2 - 6.118t)^2 + (-18.332t^2 + 8.019t)^2} \qquad (5)$$

To derive this equation, we use the distance formula in vector format and insert the 12 to convert from feet to inches. Now that we have a break function, we can find the maximum break value and at what time it occurs. We used the TI-89 calculator to calculate these values.

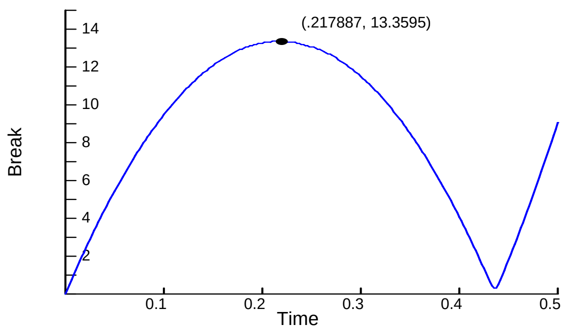


Figure 3

The numerical max as shown on the graph is about 13 inches, occurring at t = 0.218 seconds. In terms of the flight of the ball, this means the value of the break (the distance that the pitch is deviating) increases from 0 at the release of the ball until it reaches 13 inches at t = 0.218 seconds, then decreases to 0 again at t = 0.438 seconds, the time at which the ball is caught. The reason the graph continues to go up after 0.438 seconds is that this graph models a pitch in which the ball is not caught and also because $b(t)$ is defined as an absolute value function in equation 5.

Curvature

Curvature is defined as the rate of change of the unit tangent vector with respect to arc length. Intuitively, this can be thought of as the rate at which the curve changes direction at any point. Curvature can be expressed mathematically by the following equation:

$$\kappa(t) = \frac{\left|\mathbf{r}'(t) \times \mathbf{r}''(t)\right|}{\left|\mathbf{r}'(t)\right|^3}$$

To find this, we need the first and second derivatives of the position equation for our pitch. These can be found simply by differentiating the original position function (Equation 3).

Using the TI-89 computer algebra system, we insert the data for our pitch into the general curvature definition, giving us a function for the curvature of the pitch at any time:

$$\kappa(t) = .0429\,(t^2 - 2.955t + 6.227)^{(-3/2)} \qquad (6)$$

We are most concerned with the time at which the maximum curvature occurs, rather than the value. The actual unit values of the curvature function have little meaning for our purposes. Therefore, to analyze the change in the values; that is to say, to find minima, maxima, and intervals of increase and decrease, we sketch a graph of Equation 6 and numerically calculate the maximum to occur at 1.4775 seconds, using the TI-89 calculator.

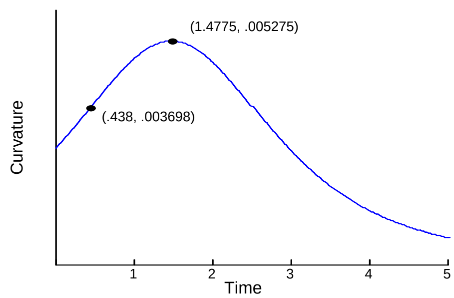


Figure 4

As we stated earlier, the actual time of the pitch (defined as the time from when it is released until the time it is either caught by the catcher or hit by the batter) is only .438 seconds. Therefore, the time we found the maximum curvature to occur (1.4775 seconds) is irrelevant for our purposes, since Equation 6 is defined on the domain [0, 0.438]. Because the curvature is strictly increasing on that domain, the maximum curvature of the pitch occurs at the final time, t = 0.438 seconds.

Thus, we find that the maximum curvature and the maximum break are not equivalent, nor do they occur at the same time. There is, however, a relationship between the two.

Curvature vs. Break

MLB.com's *GameDay* also gives the maximum break length of each pitch, the same value that we calculated for our particular pitch with our own function. To compare break and curvature, we analyzed several pitches with varying break length thrown by Matsuzaka during the same game. The original pitch we used had the greatest break, so we will also analyze the pitch with the smallest break, and three pitches in between.

Although we know that the maximum curvature really would not occur until after the pitch had been caught, we will now analyze what would happen if the pitch were allowed to keep moving without the catcher or the ground stopping it. This might seem insignificant, but recall how many facts and relationships among real numbers were not understood or even recognized until the more extensive field of complex numbers was taken into account. Analogously, we look at the hypothetical time after the pitch is caught in hopes of finding more information on the actual pitch that would be unobtainable looking at the pitch alone. We sketch a graph of the curvature of the pitches versus time (Figure 5), and then compare the maximum break length with the time of maximum curvature in the data table (Figure 6).

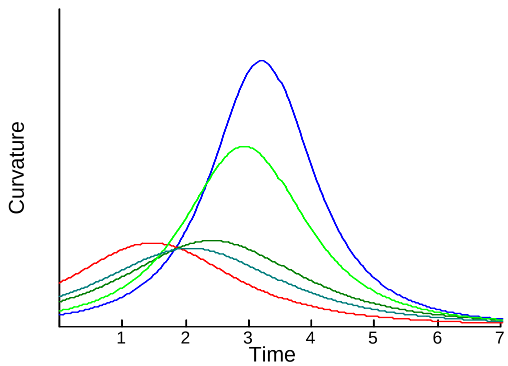


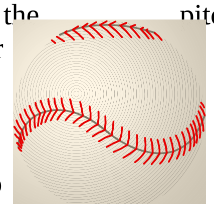
Figure 5

Maximum Break (in)	Time of Max. Curvature (sec)
3.1	3.2175
4.2	2.9435
6.5	2.4285
7.7	2.0805
14.2	1.4775

Figure 6

As can be seen on Figures 5 and 6, the pitches that reach their maximum curvature earliest have the largest break. The graph whose maximum occurs at the latest time, that is, rightmost, has the smallest maximum break at 3.1 inches, the graph where the max is leftmost has the greatest max break at 14.2 inches, and the graphs in between follow the same pattern.

So, in the example illustrated in Figures 5 and 6, we found that the pitches with the smallest break reach their maximum curvature latest, while the pitches with the greatest break reach their maximum curvature earliest. In other words, we can conclude from our data that as a pitch's maximum break value increases, the time at which maximum curvature occurs decreases.



Because the actual time of a pitch is so small, the effect of curvature on break is not evident, since the curvature is never allowed to reach its maximum. But when analyzing the curvature beyond the time parameters, we can see the effect of curvature on break. For a pitch to have a larger break, its curvature must reach its maximum relatively quickly. Meanwhile, a pitch whose maximum curvature occurs later will have a smaller break.

So, why would a batter rather face a pitch with a smaller break? The knowledgeable batter would want a pitch with a smaller break because the maximum curvature of such a pitch would not be reached until the latest time. Therefore, such a pitch would not be curving very much when it reaches the batter, making it easier to swing away. Any batter would certainly be happy with this scenario.

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